(1)

II B.Sc Maths
I shift (B/
unit - I

A RANDOM VARIABLE

A random variable is a real valued function defined on sample space (S) and taking the values in the real line $R(-\infty, \infty)$

Sample Space.

The set of all possible outcomes in a random experiment is known as sample space. It is defined by S. EXAMPLE:- If a coin is tossed once, the sample space $S = \{H, T\}$

If a die is thrown once, the sample space $S = \{1, 2, 3, 4, 5, 6\}$

PROPERTIES OF RANDOM VARIABLE:-

i) If X_1 & X_2 are random variables and c is a constant then cX_1, $X_1 + X_2$, $X_1 - X_2$ and $X_1 . X_2$ are also random variable.

ii) If X_1 & X_2 are random variables and C_1, C_2 are constants then $C_1X_1 + C_2X_2$ is also random variable

iii) If X is a random variable then $1/X$ is also a random variable

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iv) If x is a random variable then $|x|$ is also a random variable

v) If x_1 & x_2 are random variables, then $Max\{x_1, x_2\}$ & $Min\{x_1, x_2\}$ are also random variables.

vi) If x is a random variable and $f(\cdot)$ is a continuous function then $f(x)$ is a random variable.

vii) If x is a random variable and $f(\cdot)$ is an increasing function then $f(x)$ is a random variable.

DISCRETE RANDOM VARIABLE :- Countable value
(Σ)

A real valued function defined on a discrete sample space is called discrete random variable.

EXAMPLE :- Number of accidents per month, Number of success in n- trials.

PROBABILITY MASS FUNCTION :- (Pmf)

If x is a discrete random variable with distinct values $x_1, x_2, \dots x_n$, then the following conditions are satisfied

i) $P(x) = P[X = x_i] = p_i \; \forall \; x = x_i$

ii) $P(x) \geq 0 \; \forall \; x$

($\forall$ = for all values of x)

iii) $\sum_{i=1}^{\infty} P(x_i) = 1$

CONTINUOUS RANDOM VARIABLE:- $(\int)^{\text{Integral}}$

A random variable X is said to be continuous if it takes all possible values between the certain limits.

EXAMPLE:- Age, height, weight, temperature recorded per week

PROBABILITY DENSITY FUNCTION:- (Pdf)

A function $f(x)$ is said to be the pdf of a continuous random variable X if the following conditions are satisfied.

i) $f(x) \geq 0 \; \forall \; x$

ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

PROBLEMS:-

1)

x	0	1	2	3	4	5	6	7
$p(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

<i> Find k <ii> Evaluate $P[x<6]$

<iii> $P[x \geq 6]$ <iv> $P[0<x<5]$

<v> $P[x \leq 3]$ <vi> $P[2 \leq x \leq 6]$

Sol

i) $\sum_{i=1}^{\infty} P(x_i) = 1$

$$0+k+2k+2k+3k+k^2+2k^2+7k^2+k=1$$

$$9k+10k^2=1$$

$$10k^2+9k-1=0$$

$$10k^2+10k-k-1=0$$

$$10k(k+1)-(k+1)=0$$

$$(10k-1)(k+1)=0$$

$$10k=1 \quad , k=-1$$

$$k=\frac{1}{10} \quad , -1$$

k should not be negative so we take $k=\frac{1}{10}$

$$\therefore k = 1/10$$

x	0	1	2	3	4	5	6	7
$p(x)$	0	$1/10$	$2/10$	$2/10$	$3/10$	$1/100$	$2/100$	$\frac{7}{100}+\frac{1}{10}$

ii) $P[x<6]$

$$P[x<6] = P(x=0)+P(x=1)+P(x=2)+P(x=3)$$
$$+P(x=4)+P(x=5)$$

$$= 0+K+2K+2K+3K+K^2$$

$$= 8K+K^2$$

$$= 8\times\frac{1}{10}+\left(\frac{1}{10}\right)^2$$

$$= \frac{8}{10}+\frac{1}{100}$$

$$= \frac{80+1}{100}$$

$$= \frac{81}{100}$$

$$P[x<6] = 0.81$$

iii) $P[x\geq 6]$

$$P[x\geq 6] = P(x=6)+P(x=7)$$

$$= 2K^2+7K^2+K$$

$$= K+9K^2$$

$$= \frac{1}{10}+9\left(\frac{1}{100}\right)$$

$$= \frac{10+9}{100}$$

$$= \frac{19}{100}$$

$$P[x\geq 6] = 0.19$$

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iv) $P[0 < x < 5]$

$$P[0 < x < 5] = P(x=1) + P(x=2) + P(x=3) + P(x=4)$$

$$= k + 2k + 2k + 3k$$

$$= 8k$$

$$= 8(1/10) = 8/10$$

$$P[0 < x < 5] = 0.8$$

v) $P[x \leq 3]$

$$P[x \leq 3] = P(x=0) + P(x=1) + P(x=2) + P(x=3)$$

$$= 0 + k + 2k + 2k$$

$$= 5k = 5(1/10)$$

$$P[x \leq 3] = 0.5$$

vi) $P[2 \leq x \leq 6]$

$$P[2 \leq x \leq 6] = P(x=2) + P(x=3) + P(x=4) + P(x=5) + P(x=6)$$

$$= 2k + 2k + 3k + k^2 + 2k^2$$

$$= 7k + 3k^2$$

$$= 7(1/10) + 3 \times \frac{1}{100}$$

$$= \frac{7}{10} + \frac{3}{100}$$

$$= \frac{70+3}{100}$$

$$= \frac{73}{100}$$

$$P[2 \leq x \leq 6] = 0.73$$

2. A random variable x has the following probability function.

x	-2	-1	0	1	2	3
P(x)	0.1	k	0.2	2k	0.3	3k

Find (i) k (ii) $P[x < 2]$

(iii) $P[-2 < x < 2]$ (iv) $P[x \geq 2]$

Sol.

i) $\Sigma p(x_i) = 1$

$$0.1 + k + 0.2 + 2k + 0.3 + 3k = 1$$

$$0.6 + 6k = 1$$

$$6k = 1 - 0.6$$

$$6k = 0.4$$

$$k = \frac{0.4}{6}$$

$$k = 0.0667$$

ii) $P[x < 2]$

$$P[x < 2] = P[x = -2] + P[x = -1] + P[x = 0] + P[x = 1]$$

$$= 0.1 + k + 0.2 + 2k$$

$$= 0.3 + 3k$$

$$= 0.3 + 3(0.0677)$$

$$= 0.3 + 0.2001$$

$$P[x < 2] = 0.5001$$

iii) $P[-2 < x < 2]$

$P[-2 < x < 2] = P[x=-1] + P[x=0] + P[x=1]$

$= k + 0.2 + 2k$

$= 3k + 0.2$

$= 3(0.0667) + 0.2$

$= 0.2001 + 0.2$

$P[-2 < x < 2] = 0.4001$

iv) $P[x \geq 2]$

$P[x \geq 2] = P[x=2] + P[x=3]$

$= 0.3 + 3k$

$= 0.3 + 3(0.0667)$

$= 0.3 + 0.2001$

$P[x \geq 2] = 0.5001$

3.

x	0	1	2	3	4	5	6	7	8
p(x)	a	3a	5a	7a	9a	11a	13a	15a	17a

Find (i) a (ii) $P[x < 3]$

(iii) $P[x \geq 3]$ (iv) $P[0 < x < 5]$

(v) $P[x \leq 5]$ (vi). $P[x \geq 7]$

Sol.

i) $\Sigma p(x_i) = 1$

$\sum P(x) = 1$

$a + 3a + 5a + 7a + 9a + 11a + 13a + 15a + 17a = 1$

$81 \cancel{77} a = 1$

$a = \frac{1}{77}$

$a = \cancel{0.0130}$ 0.0123

ii) $P[x < 3]$

$P[x < 3] = P[x = 0] + P[x = 1] + P[x = 2]$

$= a + 3a + 5a$

$= 9a$

$= 9(0.0130)$

$= \cancel{0.117}$ 0.1107

iii) $P[x \geq 3]$

$P[x \geq 3] = P(x = 3) + P(x = 4) + P(x = 5) + P[x = 6] + P(x = 7) + P(x = 8)$

$= 7a + 9a + 7a + 13a + 15a + 17a$

$= 68a$

$= 68(0.0130)$

$= \cancel{0.884}$ 0.1968

iv) $P[0 < x < 5]$

$P[0 < x < 5] = P(x = 1) + P(x = 2) + P(x = 3) + P(x = 4)$

$= 3a + 5a + 7a + 9a$

$= 24a$

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$= 24a$

$= 24(0.0130)$

$P[0 < x < 5] = \cancel{0.312}\ 0.2952$

v) $P[x \le 5]$

$P[x \le 5] = P(x=0) + P(x=1) + P(x=2) + P(x=3)$

$\quad + P(x=4) + P(x=5)$

$= a + 3a + 5a + 7a + 9a + 7a$

$= 32a$

$= 32(0.0130)$

$P[x \le 5] = \cancel{0.416}\ 0.4428$

vi) $P[x \ge 7] = P[x=7] + P[x=8]$

$= 15a + 17a$

$= 32a$

$= 32(0.0130)$

$P[x \ge 7] = \cancel{0.416}\ 0.3936$

4. The random variable x has the following probability function.

X	−2	−1	0	1	2	3	
P(x)	0.5	0.2K	3K	0.1	K	0.3K	

(i) Find K (ii) $P[-1 < x < 2]$

(iii) $P[x \ge 2]$

~~(iv)~~

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such that $P[x \geq 1] > 0.36$

Sol

i) $\Sigma P(x_i) = 1$

$$0.5 + 0.2k + 3k + 0.1 + k + 0.3k = 1$$

$$0.6 + 4.5k = 1$$

$$4.5k = 1 - 0.6$$

$$4.5k = 0.4$$

$$k = \frac{0.4}{4.5}$$

$$k = 0.0889$$

ii) $P[-1 < x < 2]$

$$P[-1 < x < 2] = p(x=0) + p(x=1)$$

$$= 3k + 0.1$$

$$= 3(0.0889) + 0.1$$

$$= 0.2667 + 0.1$$

$$= 0.3667$$

iii) $P[x \geq 2]$

$$P[x \geq 2] = p(x=2) + p(x=3)$$

$$= k + 0.3k$$

$$= 1.3k$$

$$= 1.3(0.0889)$$

$$= 0.1156$$

iv) $P[x \geq 1] \geq 0.36$

$$P[x \geq 1] = P(x=1) + P(x=2) + P(x=3)$$

$$= 0.1 + k + 0.3k$$

$$= 1.3k + 0.1$$

$$= 1.3(0.0889) + 0.1$$

$$= 0.1156 + 0.1$$

$$= 0.2156 < 0.36$$

$\Rightarrow$ 0.2156 is the maximum value.

5. Given that

$$P(x) = \begin{cases} \frac{1}{4}, & \text{if } x = -2 \\ \frac{1}{4}, & \text{if } x = 0 \\ 1/2, & \text{if } x = 5 \\ 0, & \text{Otherwise} \end{cases}$$

i) $P[x \leq 0]$ ii) $P[x < 0]$

iii) $P(|x| > 2)$ iv) $P[0 \leq x \leq 10]$

v) $P(|x| < 2)$

Sol

i) $P[x \leq 0]$

$$P[x \leq 0] = P(x=0) + P(x=-2)$$

$$= \frac{1}{4} + \frac{1}{4}$$

$$= \frac{2}{4}$$

$$= 1/2 = 0.5$$

ii) $P[x < 0]$

$P[x < 0] = P(x = -2)$

$= \frac{1}{4}$

$= 0.25$

iii) $P[|x| > 2]$

$P[|x| > 2] = P(x = 5)$

$= \frac{1}{2}$

$= 0.5$

iv) $P[0 \leq x \leq 10]$

$P[0 \leq x \leq 10] = P(x = 0) + P(x = 5)$

$= \frac{1}{4} + \frac{1}{2}$

$= \frac{1+2}{4}$

$= \frac{3}{4}$

$= 0.75$

v) $P[|x| < 2]$ $\quad$ $|x|$ = only +ve

$P[|x| < 2] = P(x = 0)$

$= \frac{1}{4}$

$= 0.25$

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6. Given that

$$P(x) = \begin{cases} k/6 , & x=0 \\ k/3 , & x=2 \\ k/2 , & x=4 \\ 0 , & \text{otherwise} \end{cases}$$

Find k.

Sol

$$\Sigma P(x_i) = 1$$

$$\frac{k}{6} + \frac{k}{3} + \frac{k}{2} = 1$$

$$\frac{k+2k+3k}{6} = 1$$

$$\frac{6k}{6} = 1$$

$$\boxed{k=1}$$

7. A continuous random variable has pdf $f(x) = \frac{1}{18}(3+2x) ; \; 2 \le x \le 4$

Check whether it is a pdf and also find $P[2 \le x \le 3]$

Sol.

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_2^4 \frac{1}{18}(3+2x)\,dx = \frac{1}{18}\int_2^4 (3+2x)\,dx$$

$$= \frac{1}{18}\left[3x + \frac{2x^2}{2}\right]_2^4$$

$$= \frac{1}{18}\left[3x + x^2\right]_2^4$$

$$= \frac{1}{18}\left[12+16-(6+4)\right]$$

$$= \frac{1}{18}\left[28-10\right]$$

$$= \frac{18}{18}$$

$$= 1$$

Therefore, the given function is a pdf

ii) $P[2 \le x \le 3]$

$$= \int_2^3 \frac{1}{18}(3+2x)\,dx$$

$$= \frac{1}{18}\int_2^3 (3+2x)\,dx$$

$$= \frac{1}{18}\left[3x + \frac{2x^2}{2}\right]_2^3$$

$$= \frac{1}{18}\left[3x + x^2\right]_2^3$$

$$= \frac{1}{18}\left[9+9-(6+4)\right]$$

$$= \frac{1}{18}\left[18-10\right]$$

$= \frac{1}{18}[18-10]$

$= \frac{8}{18}$

$= 0.4444$

$\therefore P[2 \leq x \leq 3] = 0.4444$

8. Show that the following functions are pdf.

i) $f(x) = 6x(1-x); \quad 0 \leq x \leq 1$

ii) $f(x) = \frac{3}{4}x(2-x); \quad 0 \leq x \leq 2$

Sol

i) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\int_{-\infty}^{\infty} f(x)\,dx = \int_0^1 6x(1-x)\,dx$

$= 6\int_0^1 (x - x^2)\,dx$

$= 6\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$

$= 6\left[\frac{1}{2} - \frac{1}{3} - 0 - 0\right]$

$= 6\left[\frac{1}{2} - \frac{1}{3}\right]$

$= 6\left[\frac{3-2}{6}\right]$

$= \frac{6(1)}{6} = 1$

$\therefore$ The Given function is a pdf.

ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{0}^{2} \frac{3}{4} x(2-x)\,dx$$

$$= \frac{3}{4}\int_{0}^{2}(2x - x^2)\,dx$$

$$= \frac{3}{4}\left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^2$$

$$= \frac{3}{4}\left[x^2 - \frac{x^3}{3}\right]_0^2$$

$$= \frac{3}{4}\left[4 - \frac{8}{3}\right]$$

$$= \frac{3}{4}\left[\frac{12-8}{3}\right]$$

$$= \frac{3}{4}\left[\frac{4}{3}\right]$$

$$\therefore \int_{0}^{2} \frac{3}{4} x(2-x)\,dx = 1$$

$\therefore$ The Given function is a pdf.

9. Find the constants for the following pdf functions.

$f(x) = Ax(2-x)$, $0 \le x \le 2$

$f(x) = k(2-x)$, $0 \le x \le 2$

Sol

i) $f(x) = Ax(2-x)$; $0 \le x \le 2$

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$f(x) = Ax(2-x); \quad 0 \le x \le 2$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_0^2 Ax(2-x)\,dx = 1$$

$$A\int_0^2 (2x - x^2)\,dx = 1$$

$$A\left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^2 = 1$$

$$A\left[4 - \frac{8}{3}\right] = 1$$

$$A\left[\frac{12-8}{3}\right] = 1$$

$$A\left[\frac{4}{3}\right] = 1$$

$$A = \frac{3}{4}$$

The value of the constant A is $\frac{3}{4}$

ii) $f(x) = K(2-x), \quad 0 \le x \le 2$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_0^2 K(2-x)\,dx = 1$$

$$K\int_0^2 (2-x)\,dx = 1$$

$$K\left[2x - \frac{x^2}{2}\right]_0^2 = 1$$

$$K\left[2(2) - \frac{(2)}{2} - 0\right] = 1$$

$$K[4-2] = 1$$

$$2K = 1$$

$$K = \frac{1}{2}$$

The value of the constant K is $\frac{1}{2}$

10. A continuous random variable has pdf. $f(x) = \frac{1}{x^2}$

Find i) $P[1 \le x \le 2]$

ii) $P[4 \le x \le 5]$

<u>Sol</u>.

$$f(x) = \frac{1}{x^2}$$

i) $P[1 \le x \le 2]$

$$\Rightarrow \int_1^2 \frac{1}{x^2}\,dx$$

$$= \int_1^2 x^{-2}\,dx$$

$$= \left[\frac{x^{-2+1}}{-2+1}\right]_1^2$$

$$= \left[\frac{x^{-1}}{-1}\right]_1^2$$

$$= -\left[\frac{1}{x}\right]_1^2$$

$$= -\left[\frac{1}{x}\right]_1^2$$

$$= -\left[\frac{1}{2} - \frac{1}{1}\right]$$

$$= -\frac{1}{2} + 1$$

$$= \frac{1}{2}$$

$P[1 \leq x \leq 2] = 1/2$

ii) $P[4 \leq x \leq 5]$

$$\Rightarrow \int_4^5 \frac{1}{x^2}\, dx$$

$$= \int_4^5 x^{-2}\, dx$$

$$= \left[\frac{x^{-2+1}}{-2+1}\right]_4^5$$

$$= \left[\frac{x^{-1}}{-1}\right]_4^5$$

$$= -\left[\frac{1}{5} - \frac{1}{4}\right]$$

$$= -1\left[\frac{4-5}{20}\right]$$

$$= -1\frac{(-1)}{20}$$

$$= \frac{1}{20}$$

$\therefore\ P[4 \leq x \leq 5] = \frac{1}{20}$

A continuous random variable X having

pdf. $f(x) = \begin{cases} 3x^2 & , 0 < x < 1 \\ 0 & , \text{Otherwise} \end{cases}$

Verify it is a pdf & also find

i) $P[X \leq 1/3]$

ii) $P[\frac{1}{3} \leq X \leq \frac{1}{2}]$

iii) $P[X \leq 1/2 / 1/3 \leq X \leq 2/3]$

Sol.

i) $f(x) = 3x^2 \quad , 0 < x < 1$

$$\int_0^1 3x^2 \, dx$$

$$3 \int_0^1 x^2 \, dx$$

$$3 \left[\frac{x^3}{3}\right]_0^1$$

$$\frac{3}{3}[1-0] = 1$$

$\therefore$ The Given function is a pdf

i) $P[X \leq 1/3]$

$$\int_0^{1/3} 3x^2 \, dx = 3 \int_0^{1/3} x^2 \, dx$$

$$= 3 \left[\frac{x^3}{3}\right]_0^{1/3}$$

$$= \frac{3}{3}\left[\left(\frac{1}{3}\right)^3 - 0\right]$$

$$= \frac{1}{27} = 0.0370$$

ii) $P\left[\frac{1}{3} \le x \le \frac{1}{2}\right]$

$$\int_{1/3}^{1/2} 3x^2\,dx = 3\left[\frac{x^3}{3}\right]_{1/3}^{1/2}$$

$$= \frac{3}{3}\left[\left(\frac{1}{2}\right)^3 - \left(\frac{1}{3}\right)^3\right]$$

$$= \left[\frac{1}{8} - \frac{1}{27}\right]$$

$$= \frac{27-8}{216}$$

$$= \frac{19}{216}$$

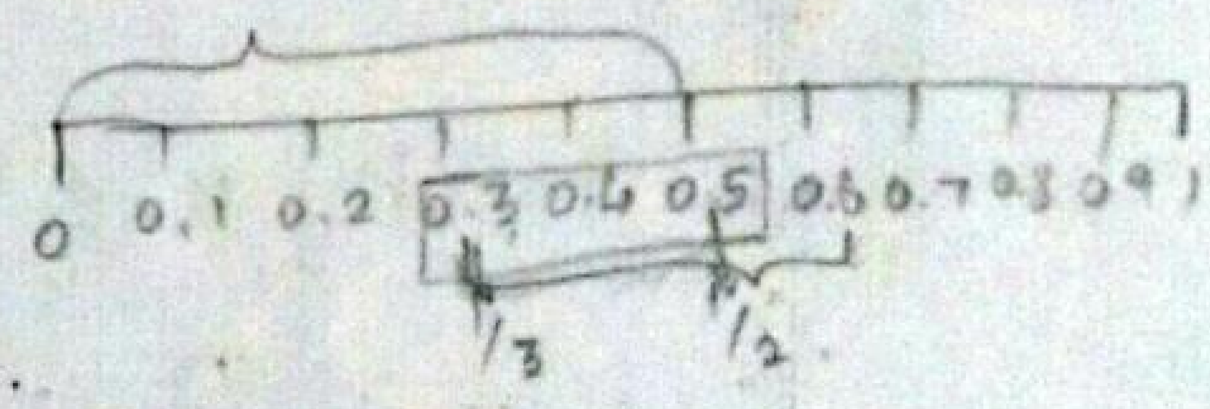
$P\left[\frac{1}{3} \le x \le \frac{1}{2}\right] = 0.0880$

iii) $P\left[x \le \frac{1}{2} / \left(\frac{1}{3} \le x \le \frac{2}{3}\right)\right]$

$$P(X/Y) = \frac{P(X \cap Y)}{P(Y)}$$

$$P\left[x \le \frac{1}{2} / \frac{1}{3} \le x \le \frac{2}{3}\right] = \frac{P\left[\left(x \le \overset{0.5}{\frac{1}{2}}\right) \cap \left(\overset{0.3}{\frac{1}{3}} \le x \le \overset{0.6}{\frac{2}{3}}\right)\right]}{P\left[\frac{1}{3} \le x \le \frac{2}{3}\right]}$$

$$= \frac{P\left[\frac{1}{3} \le x \le \frac{1}{2}\right]}{P\left[\frac{1}{3} \le x \le \frac{2}{3}\right]}$$



$$P\left[\frac{1}{3} \le x \le \frac{2}{3}\right] = \int_{1/3}^{2/3} 3x^2\,dx$$

$$= 3\left[\frac{x^3}{3}\right]_{1/3}^{2/3}$$

$$= \left[\frac{8}{27} - \frac{1}{27}\right]$$

$= \frac{1}{27}$

$P\left[\frac{1}{3} \leq x \leq \frac{2}{3}\right] = 0.2593$

$P\left[\frac{1}{3} \leq x \leq \frac{1}{2}\right] : 0.088$

$$\frac{P\left[\frac{1}{3} \leq x \leq \frac{1}{2}\right]}{P\left[\frac{1}{3} \leq x \leq \frac{2}{3}\right]} = \frac{0.088}{0.2593}$$

$$= 0.3394.$$

DISTRIBUTION FUNCTION OR CUMULATIVE DISTRIBUTION FUNCTION

Let x be a random variable with function F defined for all real values of x by $F(x) = P[X \leq x]; -\infty < x < \infty$ is called the distribution function of the random variable X.

PROPERTIES OF DISTRIBUTION FUNCTION:-

i) $F(-\infty) = 0$

ii) $F(\infty) = 1$

iii) $P[a \leq x \leq b] = F(b) - F(a)$

DISCRETE DISTRIBUTION FUNCTION

Let X be a discrete random variable. The function F is said to be discrete distribution function if

$F(x) = P[X \leq x]; -\infty < x < \infty$ and

$F(x) = \sum_{i=1}^{\infty} P(x_i) = 1$

CONTINUOUS DISTRIBUTION FUNCTION:

Let X be a continuous random variable with pdf then the function

$F(x) = P[X \leq x]$

$= \int_{-\infty}^{\infty} f(t)\,dt \; ; -\infty < x < \infty$ is called

continuous distribution function of the random variable X.

PROPERTIES OF CONTINUOUS DISTRIBUTION FUNCTION:-

i) $0 \leq F(x) \leq 1 \; ; -\infty < x < \infty$

ii) $F(x)$ is non-decreasing function of x

iii) $F(-\infty) = \int_{-\infty}^{-\infty} f(x)\,dx = 0$

iv) $F(\infty) = \int_{-\infty}^{\infty} f(x)\,dx = 1$

v) $F(x)$ is a constant function of x on the right

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vi) The discontinuities of $F(x)$ are at the most countable.

vii) $P[a \le x \le b] = F(b) - F(a)$

viii) $F'(x) = \frac{d}{dx} F(x) = f(x)$

$\Rightarrow dF(x) = f(x)dx$

PROBLEM:-

1. From the problem (1) - (Discrete)

x	0	1	2	3	4	5	6	7
$p(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

vii) If $P[x \le a] > \frac{1}{2}$. Find the maximum value of a.

viii) Determine the distribution function of x

Sol.

$k = \frac{1}{10}$

viii) $F(x) = P[X \le x]$

$P[X \le 0] = P[X = 0]$

$= 0$

$P[X \le 1] = P[X=0] + P[X=1]$

$= 0 + k$

$= 0 + \frac{1}{10}$

$= \frac{1}{10}$

$$P[x \leq 2] = P[x=0] + P[x=1] + P[x=2]$$

$$= 0 + k + 2k$$

$$= 3k$$

$$= 3\left(\frac{1}{10}\right)$$

$$= \frac{3}{10}$$

$$P[x \leq 3] = P[x=0] + P[x=1] + P[x=2] + P[x=3]$$

$$= 0 + k + 2k + 2k$$

$$= 5k$$

$$= \frac{5}{10}$$

$$= \frac{1}{2}$$

$$P[x \leq 4] = P[x=0] + P[x=1] + P[x=2] + P[x=3] + P[x=4]$$

$$= 0 + k + 2k + 2k + 3k$$

$$= 8k$$

$$= \frac{8}{10}$$

$$= \frac{4}{5}$$

$$P[x \leq 5] = P[x=0] + P[x=1] + P[x=2] + P[x=3] + P[x=4] + P[x=5]$$

$$= 0 + k + 2k + 2k + 3k + k^2$$

$$= 8k + k^2$$

$= \frac{8}{10} + \frac{1}{100}$

$= \frac{80+1}{100}$

$= \frac{81}{100}$

$P[x \leq 6] = P[x=0] + P[x=1] + P[x=2] + P[x=3] +$

$P[x=4] + P[x=5] + P[x=6]$

$= 0 + k + 2k + 2k + 3k + k^2 + 2k^2$

$= 8k + 3k^2$

$= \frac{8}{10} + \frac{3}{100}$

$= \frac{80+3}{100}$

$= \frac{83}{100}$

$P[x \leq 7] = P[x=0] + P[x=1] + P[x=2] +$

$P[x=3] + P[x=4] + P[x=5] +$

$P[x=6] + P[x=7]$

$= 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k$

$= 9k + 10k^2$

$= \frac{9}{10} + \frac{10}{100}$

$= \frac{10}{10}$

$= 1$

$$F(x=) = \begin{cases} 0, & 0 \le x < 1 \\ \frac{1}{10}, & 1 \le x < 2 \\ \frac{3}{10}, & 2 \le x < 3 \\ \frac{1}{2}, & 3 \le x < 4 \\ \frac{4}{5}, & 4 \le x < 5 \\ \frac{81}{100}, & 5 \le x < 6 \\ \frac{83}{100}, & 6 \le x < 7 \\ 1, & x \ge 7 \end{cases}$$

viii) By trial, we get $a = 4$

2.

x	0	1	2	3	4
$p(x)$	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{2}{7}$	$\frac{1}{7}$	$\frac{1}{7}$

Find the distribution function

Sol

$$F(x) = P[X \le x]$$

$$P[X \le 0] = P[X = 0]$$

$$= \frac{1}{7}$$

$$P[X \le 1] = P[X = 0] + P[X = 1]$$

$$= \frac{1}{7} + \frac{2}{7}$$

$$= \frac{3}{7}$$

$$P[x \leq 2] = P[x=0] + P[x=1] + P[x=2]$$

$$= \frac{1}{7} + \frac{2}{7} + \frac{2}{7}$$

$$= \frac{5}{7}$$

$$P[x \leq 3] = P[x=0] + P[x=1] + P[x=2] + P[x=3]$$

$$= \frac{1}{7} + \frac{2}{7} + \frac{2}{7} + \frac{1}{7}$$

$$= \frac{6}{7}$$

$$P[x \leq 4] = P[X=0] + P[x=1] + P[x=2] + P[x=3] + P[x=4]$$

$$= \frac{1}{7} + \frac{2}{7} + \frac{2}{7} + \frac{1}{7} + \frac{1}{7}$$

$$= \frac{7}{7}$$

$$= 1$$

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{7}, & 0 \leq x < 1 \\ \frac{3}{7}, & 1 \leq x < 2 \\ \frac{5}{7}, & 2 \leq x < 3 \\ \frac{6}{7}, & 3 \leq x < 4 \\ 1, & x \geq 4 \end{cases}$$

Let x be a continuous random variable with pdf.

$$f(x) = \begin{cases} kx , & 0 \leq x < 1 \\ k , & 1 \leq x < 2 \\ -kx+3k , & 2 \leq x < 3 \\ 0 , & \text{Otherwise} \end{cases}$$

i) Determine the constant k.

ii) Determine the cumulative distribution function.

Sol

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\int_0^1 kx\,dx + \int_1^2 k\,dx + \int_2^3 (-kx+3k)\,dx = 1$$

$$\left[k\frac{x^2}{2}\right]_0^1 + \left[kx\right]_1^2 + \left[-\frac{kx^2}{2} + 3xk\right]_2^3 = 1$$

$$k\left[\left(\frac{1}{2} - 0\right) + (2-1) + \left(\frac{-9}{2} + 9 - \left(-\frac{4}{2} + 6\right)\right)\right] = 1$$

$$k\left[\frac{1}{2} + 1 + \left(\frac{-9+18}{2} - (4)\right)\right] = 1$$

$$\frac{-4+12}{2}.$$

$$-\frac{8}{2} = -4.$$

$$k\left[\frac{1}{2} + 1 + \frac{9}{2} - 4\right] = 1$$

$$k\left[\frac{10}{2} - 3\right] = 1$$

$$k\left[5-3\right] = 1$$

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$2k = 1$

$k = \frac{1}{2}$

ii) $$F(x) = \int_{-\infty}^{x} f(t)\,dt$$

For any x,

$$-\infty \leq x < 0$$

$$F(x) = \int_{-\infty}^{x} 0\,dt$$

$$= 0$$

For any x, $0 \leq x < 1$

$$F(x) = \int_{-\infty}^{0} f(t)\,dt + \int_{0}^{x} f(t)\,dt$$

$$= 0 + \int_{0}^{x} \frac{1}{2}\,t\,dt$$

$$= \frac{1}{2}\left[\frac{t^2}{2}\right]_0^x$$

$$= \frac{x^2}{4}$$

For any x, $1 \leq x < 2$

$$F(x) = \int_{-\infty}^{0} f(t)\,dt + \int_{0}^{1} f(t)\,dt + \int_{1}^{x} f(t)\,dt$$

$$= 0 + \int_{0}^{1} kt\,dt + \int_{1}^{x} k\,dt$$

$$= \frac{1}{2}\int_{0}^{1} t\,dt + \frac{1}{2}\int_{1}^{x} dt$$

$$= \frac{1}{2}\left[\frac{t^2}{2}\right]_0^1 + \frac{1}{2}\left[t\right]_1^x$$

$$= \frac{1}{2}\left[\frac{1}{2} - 0\right] + \frac{1}{2}\left[x - 1\right]$$

$$= \frac{1}{4} - \frac{1}{2} + \frac{x}{2}$$

$$= \frac{1-2}{4} + \frac{x}{2}$$

$$= \frac{x}{2} - \frac{1}{4}$$

For any x, $2 \le x < 3$

$$F(x) = \int_{-\infty}^{0} f(t)dt + \int_{0}^{1} f(t)dt + \int_{1}^{2} f(t)dt + \int_{2}^{x} f(t)dt$$

$$= 0 + \int_{0}^{1} kt\,dt + \int_{1}^{2} k\,dt + \int_{2}^{x} (-kt + 3k)dt$$

$$= \frac{1}{2}\left[\frac{t^2}{2}\right]_0^1 + \frac{1}{2}\left[t\right]_1^2 + k\left[-\frac{t^2}{2} + 3t\right]_2^x$$

$$= \frac{1}{2}\left[\frac{1}{2}\right] + \frac{1}{2}\left[2-1\right] + \frac{1}{2}\left[-\frac{x^2}{2} + 3x - \left(-\frac{4}{2} + 6\right)\right]$$

$$= \frac{1}{4} + \frac{1}{2} + \frac{1}{2}\left[-\frac{x^2}{2} + 3x - 4\right]$$

$$= \frac{1+2}{4} - \frac{x^2}{4} + \frac{3x}{2} - \frac{4}{2}$$

$$= \frac{3}{4} - \frac{x^2}{4} + \frac{3x}{2} - 2$$

$$= \frac{3}{4} - 2 - \frac{x^2}{4} + \frac{3x}{2}$$

$$= \frac{3-8}{4} - \frac{x^2}{4} + \frac{3x}{2}$$

$$F(x) = -\frac{5}{4} - \frac{x^2}{4} + \frac{3x}{2}$$

$$F(x) = \int_{-\infty}^{0} f(t)\,dt + \int_{0}^{1} f(t)\,dt + \int_{1}^{2} f(t)\,dt + \int_{2}^{3} f(t)\,dt + \int_{3}^{x} f(t)\,dt$$

$$= 0 + \int_0^1 \frac{1}{2} t\,dt + \int_1^2 \frac{1}{2}\,dt + \frac{1}{2}\int_2^3 (-t+3)\,dt + \int_3^x 0\,dt$$

$$= \frac{1}{2}\left[\frac{t^2}{2}\right]_0^1 + \frac{1}{2}\left[t\right]_1^2 + \frac{1}{2}\left[-\frac{t^2}{2} + 3t\right]_2^3 + 0$$

$$= \frac{1}{2}\left[\frac{1}{2} - 0\right] + \frac{1}{2}\left[2 - 1\right] + \frac{1}{2}\left[-\frac{9}{2} + 9 - \left(-\frac{4}{2} + 6\right)\right]$$

$$= \frac{1}{4} + \frac{1}{2} + \frac{1}{2}\left[-\frac{9}{2} + 9 + \frac{4}{2} - 6\right]$$

$$= \frac{1}{4} + \frac{1}{2} + \frac{1}{2}\left[-\frac{9}{2} + \frac{4}{2} + 3\right]$$

$$= \frac{1}{4} + \frac{1}{2} + \frac{1}{2}\left[-\frac{5}{2} + 3\right]$$

$$= \frac{1}{4} + \frac{1}{2} + \frac{1}{2}\left[\frac{-5+6}{2}\right]$$

$$= \frac{1}{4} + \frac{1}{2} + \frac{1}{2}\left[\frac{1}{2}\right]$$

$$= \frac{1}{4} + \frac{1}{2} + \frac{1}{4}$$

$$= \frac{1+2+1}{4}$$

$$= \frac{4}{4}$$

$$= 1$$

$$F(x) = \begin{cases} 0, & -\infty \le x < 0 \\ x^2/4, & 0 \le x < 1 \\ \frac{x}{2} - \frac{1}{4}, & 1 \le x < 2 \\ -5/4 - x^2/4 + 3x/2, & 2 \le x < 3 \\ 1, & 3 \le x < \infty \end{cases}$$

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TWO DIMENSIONAL RANDOM VARIABLE

Let X, Y be the two random variables defined on the sample space (S), then the function (X, Y) that assigns a point in R^2 is called a two dimensional random variable.

TWO DIMENSIONAL DISCRETE RANDOM VARIABLE OR JOINT pmf :-

Let X and Y be the two dimensional discrete random variable. Let $p(x_i, y_i)$ be the joint pmf of X and Y if the following conditions are satisfied.

* $P(x, y) = P[X = x_i, Y = y_j] \quad i, j = 1, 2, 3, \ldots$

* $P[x_i, y_j] \geq 0$

* $\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} P(x_i, y_j) = 1$

TWO DIMENSIONAL OR JOINT pdf :-

Let X and Y be the two dimensional continuous random variables then $f(x, y)$ is the joint pdf of x and y, if the following conditions are satisfied.

* $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y)\, dx\, dy = 1$

* $f(x,y) \geq 0 \quad \forall x, y$

MARGINAL PROBABILITY FUNCTION:-

Let X and Y be the two dimensional discrete random variables with joint pmf $P(x_i, y_j)$ then the marginal pmf of x and y are given by

$F(x) =$ dy
$F(y) =$ dx

* $p(x) = \sum_{y} P[X = x_i, Y = y_j] = \sum_{y} P(x_i, y_j)$

* $p(y) = \sum_{x} P[X = x_i, Y = y_j] = \sum_{x} P(x_i, y_j)$

Let X, Y be the two dimensional continuous random variable with pdf $f(x,y)$ then, the marginal pdf of x and y are given by

* $f(x) = \int_{-\infty}^{\infty} f(x,y)\, dy$

* $f(y) = \int_{-\infty}^{\infty} f(x,y)\, dx$

CONDITIONAL PROBABILITY FUNCTION:-

Let X and Y be the discrete random variable with joint pmf $p(x,y)$ then, the conditional pmf of X given $Y = y$ is

$$P(X/Y=y) = \frac{P[X=x, Y=y]}{P[Y=y]} = \frac{p(x,y)}{p(y)}$$

The conditional pmf of Y given X = x is

$$P(Y/X=x) = \frac{P[X=x, Y=y]}{P[X=x]} = \frac{p(x,y)}{p(x)}$$

Let X and Y be the continuous random variable with joint pdf f(x, y) then the conditional pdf of X given Y = y is

$$f(X/Y=y) = \frac{f(x,y)}{f(y)}, \quad f(y) > 0$$

The conditional pdf of Y given X = x is

$$f(Y/X=x) = \frac{f(x,y)}{f(x)}, \quad f(x) > 0$$

TWO DIMENSIONAL DISTRIBUTION FUNCTION:-

The distribution function of the two dimensional random variable X, Y is a real valued function F defined for all real values of x and y by, $F(x,y) = P[X \le x, Y \le y]$

MARGINAL DISTRIBUTION FUNCTION:-

Let $p(x,y)$ be the joint distribution function of the discrete random variables of X and Y. The marginal distribution function of X and Y are given by

$$F(x) = \sum_{y} P[X \le x, Y = y]$$

$$F(y) = \sum_{x} P[X = x, Y \le y]$$

Let $F(x,y)$ be the joint distribution function of the continuous random variables X and Y. The marginal distribution function of X and Y are given by

$$F(x) = \int_{-\infty}^{x} \left[\int_{-\infty}^{\infty} F(x,y)\, dy \right] dx$$

$$F(y) = \int_{-\infty}^{y} \left[\int_{-\infty}^{\infty} F(x,y)\, dx \right] dy$$

INDEPENDENT RANDOM VARIABLE:-

Two random variables X and Y with joint pdf [or pmf] $f(x,y)$ and marginal pdf's (pmf's) $f(x)$ and $g(y)$ respectively are said to be stochastically independent if and only if (iff)

$$f(x,y) = f(x).g(y)$$

PROBLEMS:-

1. For the joint probability distribution of two random variables X and Y

Y / X	1	2	3	4	Total p(x)
1	4/36	3/36	2/36	1/36	10/36
2	1/36	3/36	3/36	2/36	9/36
3	5/36	1/36	1/36	1/36	8/36
4	1/36	2/36	1/36	5/36	9/36.
Total p(y)	11/36	9/36	7/36	9/36	36/36

Find

i) The marginal probability distribution of X and Y.

ii) Conditional distribution of X given Y=1 and Y given X=2

Sol

The marginal probability distribution of X is

$$p(x) = P[X=x] = \sum_y P[X=x, Y=y]$$

$$P[X=1] = \sum_{y}\left[P(X=1, Y=1,2,3,4)\right]$$

$$= P[X=1, Y=1] + P[X=1, Y=2] +$$

$$P[X=1, Y=3] + P[X=1, Y=4]$$

$$= \frac{4}{36} + \frac{3}{36} + \frac{2}{36} + \frac{1}{36}$$

$$P[X=2] = P[X=2, Y=1] + P[X=2, Y=2] +$$

$$P[X=2, Y=3] + P[X=2, Y=4]$$

$$= \frac{1}{36} + \frac{3}{36} + \frac{3}{36} + \frac{2}{36}$$

$$= \frac{9}{36}$$

$$P[X=3] = \sum_{y}\left[P(X=3, Y=1,2,3,4)\right]$$

$$= P[X=3, Y=1] + P[X=3, Y=2] +$$

$$P[X=3, Y=3] + P[X=3, Y=4]$$

$$= \frac{5}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36}$$

$$= \frac{8}{36}$$

$$P(X=4] = \sum_{y}\left[P(X=4, Y=1,2,3,4)\right]$$

$$= P[X=4, Y=1] + P[X=4, Y=2] +$$

$$P[X=4, Y=3] + P[X=4, Y=4]$$

$$= \frac{1}{36} + \frac{2}{36} + \frac{1}{36} + \frac{5}{36}$$

$$= \frac{9}{36}$$

The marginal probability distribution of Y is

$$P(y) = P[Y=y] = \sum_x P[X=x, Y=y]$$

$$P[Y=1] = \sum_x [P(Y=1, X=1,2,3,4)]$$

$$= P[Y=1, X=1] + P[Y=1, X=2] + P[Y=1, X=3] + P[Y=1, X=4]$$

$$= \frac{4}{36} + \frac{1}{36} + \frac{5}{36} + \frac{1}{36}$$

$$= \frac{11}{36}$$

$$P[Y=2] = \sum_x [P(Y=2, X=1,2,3,4)]$$

$$= P[Y=2, X=1] + P[Y=2, X=2] + P[Y=2, X=3] + P[Y=2, X=4]$$

$$= \frac{3}{36} + \frac{3}{36} + \frac{1}{36} + \frac{2}{36}$$

$$= \frac{9}{36}$$

$$P[Y=3] = \sum_x [P(Y=3, X=1,2,3,4)]$$

$$= P[Y=3, X=1] + P[Y=3, X=2] + P[Y=3, X=3] + P[Y=3, X=4]$$

$$= \frac{2}{36} + \frac{3}{36} + \frac{1}{36} + \frac{1}{36}$$

$$= \frac{7}{36}$$

$P(Y=4) = \sum_{x} [P(Y=4, X=1,2,3,4)]$

$= P[Y=4, X=1] + P[Y=4, X=2] + P[Y=4, X=3] + P[Y=4, X=4]$

$= \frac{1}{36} + \frac{2}{36} + \frac{1}{36} + \frac{5}{36}$

$= \frac{9}{36}$

The marginal probability distribution of X is

x	1	2	3	4
$p(x)$	$\frac{10}{36}$	$\frac{9}{36}$	$\frac{8}{36}$	$\frac{9}{36}$

The marginal probability distribution of Y is

y	1	2	3	4
$p(y)$	$\frac{11}{36}$	$\frac{9}{36}$	$\frac{7}{36}$	$\frac{9}{36}$

ii) The conditional distribution of X/Y

$$P(x/y) = P[X=x/Y=y] = \frac{P[X=x, Y=y]}{P[Y=y]}$$

$$P[X=1/Y=1] = \frac{P[X=1, Y=1]}{P[Y=1]} = \frac{4/36}{11/36} = \frac{4}{11}$$

$$P[X=2/Y=1] = \frac{P[X=2, Y=1]}{P[Y=1]} = \frac{1/36}{11/36} = \frac{1}{11}$$

$$P[X=3/Y=1] = \frac{P[X=3, Y=1]}{P[Y=1]} = \frac{5/36}{11/36} = \frac{5}{11}$$

$$P[X=4/Y=1] = \frac{P[X=4, Y=1]}{P[Y=1]} = \frac{1/36}{11/36} = \frac{1}{11}$$

The conditional distribution of Y/X

$$P(y/x) = P[Y=y / X=x] = \frac{P[X=x, Y=y]}{P[X=x]}$$

$$P[Y=1/X=2] = \frac{P[X=2, Y=1]}{P[X=2]} = \frac{1/36}{9/36} = \frac{1}{9}$$

$$P[Y=2/X=2] = \frac{P[X=2, Y=2]}{P[X=2]} = \frac{3/36}{9/36} = \frac{3}{9} = \frac{1}{3}$$

$$P[Y=3/X=2] = \frac{P[X=2, Y=3]}{P[X=2]} = \frac{3/36}{9/36} = \frac{3}{9} = \frac{1}{3}$$

$$P[Y=4/X=2] = \frac{P[X=2, Y=4]}{P[X=2]} = \frac{2/36}{9/36} = \frac{2}{9}$$

The conditional distribution of $X/Y=1$ is

x	1	2	3	4
$P(X/Y=1)$	$4/11$	$1/11$	$5/11$	$1/11$

The conditional distribution of $Y/X=2$ is

y	1	2	3	4
$P(Y/X=2)$	$1/9$	$3/9$	$3/9$	$2/9$

Let X and Y be the two dimensional random variable with joint pdf:

$$f(x,y) = \frac{1}{8}(6-x-y), \quad 0 \le x < 2, \quad 2 \le y < 4$$

Find (i) $P[X<1 \cap Y>3]$

(ii) $P[X+Y<3]$

(iii) $P[X<1 / Y<3]$

Solution:

i) $$f(x,y) = \int_0^1 \int_2^3 \frac{1}{8}(6-x-y)\,dy\,dx$$

$$= \frac{1}{8}\left[\int_0^1 \left(6y - xy - \frac{y^2}{2}\right)_2^3 dx\right]$$

$$= \frac{1}{8}\left\{\int_0^1 \left(18-3x-\frac{9}{2}\right) - \left(12-2x-\frac{4}{2}\right) dx\right\}$$

$$= \frac{1}{8}\left\{\int_0^1 \left(18-12-3x+2x-\frac{9}{2}+\frac{4}{2}\right) dx\right\}$$

$$= \frac{1}{8}\int_0^1 \left(6-x-\frac{5}{2}\right) dx$$

$$= \frac{1}{8}\int_0^1 \left(6-x-\frac{5}{2}\right) dx$$

$$= \frac{1}{8}\left[6x - \frac{x^2}{2} - \frac{5x}{2}\right]_0^1$$

$$= \frac{1}{8}\left[6 - \frac{1}{2} - \frac{5}{2}\right]_0^1$$

$$= \frac{1}{8}\left[6 - \frac{\cancel{6}^{3}}{\cancel{2}}\right] \Rightarrow \frac{3}{8}$$

$$\therefore P[X<1 \cap Y<3] = \frac{3}{8}$$

$$(ii)\ P[X+Y<3] = \int_0^1 \int_2^{3-x} \frac{1}{8}(6-x-y)\,dy\,dx$$

$$= \frac{1}{8}\int_0^1 \left[6y - xy - \frac{y^2}{2}\right]_2^{3-x} dx$$

$$= \frac{1}{8}\int_0^1 \left[6(3-x) - x(3-x) - \frac{(3-x)^2}{2} - \left(12 - 2x - \frac{4}{2}\right)\right] dx$$

$$= \frac{1}{8}\int_0^1 \left[18 - 6x - 3x + x^2 - \frac{(9+x^2-6x)}{2} - 12 + 2x + 2\right] dx$$

$$= \frac{1}{8}\int_0^1 \left[18 - 6x - 3x + x^2 - \frac{9}{2} - \frac{x^2}{2} + \frac{6x}{2} - 12 + 2x + 2\right] dx$$

$$= \frac{1}{8}\left[18x - \frac{9x^2}{2} + \frac{x^3}{3} - \frac{9x}{2} - \frac{x^3}{6} + \frac{6x^2}{4} - 12x + \frac{2x^2}{2} + 2x\right]_0^1$$

$$= \frac{1}{8}\left[18 - \frac{9}{2} + \frac{1}{3} - \frac{9}{2} - \frac{1}{6} + \frac{3}{2} - 12 + 1 + 2\right]$$

$$= \frac{1}{8}\left[18 - 12 + 1 + 2 - \frac{9}{2} - \frac{9}{2} + \frac{3}{2} + \frac{1}{3} - \frac{1}{6}\right]$$

$$= \frac{1}{8}\left[9 - \frac{\cancel{18}^{9}}{\cancel{2}} + \frac{9+2}{6} - \frac{1}{6}\right.$$

$$= \frac{1}{8}\left[\frac{11}{6} - \frac{1}{6}\right] \Rightarrow \frac{1}{8}\left[\frac{\cancel{10}^{5}}{\cancel{6}_{3}}\right]$$

$$= \frac{5}{24}$$

$$\therefore P[X+Y<3] = \frac{5}{24}$$

(iii) $P[X<1 / Y<3] = \dfrac{P[X<1 \cap Y<3]}{P[Y<3]}$

$$P[Y<3] = \int_2^3 \int_0^2 f(x,y)\,dx\,dy$$

$$= \int_2^3 \int_0^2 \frac{1}{8}(6-x-y)\,dx\,dy$$

$$= \int_2^3 \left[\frac{1}{8}\int_0^2 6-x-y\;dx\right] dy$$

$$= \int_2^3 \frac{1}{8}\left[6x-\frac{x^2}{2}-xy\right]_0^2 dy$$

$$= \int_2^3 \frac{1}{8}\left[12-\frac{\cancel{4}^{2}}{\cancel{2}}-2y\right] dy$$

$$= \int_2^3 \frac{1}{8}(10-2y)\,dy$$

$$= \int_2^3 \frac{2}{8}(5-y)\,dy$$

$$= \frac{1}{4}\int_2^3 (5-y)\,dy$$

$$= \frac{1}{4}\left[5y-\frac{y^2}{2}\right]_2^3$$

$$= \frac{1}{4}\left[15-\frac{9}{2}-10+\frac{\cancel{4}^{2}}{\cancel{2}}\right]$$

$$= \frac{1}{4}\left[7-\frac{9}{2}\right] \Rightarrow \frac{1}{4}\left[\frac{14-9}{2}\right]$$

$$P[Y<3] = \frac{5}{8}$$

$$P[X<1 / Y<3] = \frac{P[X<1 \cap Y<3]}{P[Y<3]}$$

$$= \frac{3/8}{5/8}$$

$$P[X<1 / Y<3] = \frac{3}{5}$$